



On Performance Measure for Intermittencies in Vehicle Routing Problems (IVRP) with Road Restrictions and Forced Split Deliveries

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Abstract: Business activities are expanding daily as numerous customers enter the supply chain. The distribution, transportation, and supply chain management problems currently ravaging business activities are of different issues ranging from varying customers with other entry conditions into the market, different vehicles with various conditions and different restrictions facing the routes traversed are not left out. This paper aims to develop a model for Intermittencies in Vehicle Routing Problems (IVRP) that will holistically annex the various priorities and road restrictions leading to a forced split delivery occasioned by either the Customers' Vehicle Preference, the Road Time Restriction, the Vehicle Weight Restriction, or the Vehicle Height Restriction. It will consider the service choices and recent research updates on customers' intermittencies in vehicle routing problems as well as look into the differences in customers' demand over a vehicle carrying capacity. The paper discusses various stages of Late Request Customers (LRC) as Pre-Service stage, During Service Stage, and Post Service Stage that interposes in Early Request Customers (ERC) hence resulting in intermittencies referred to as Intermittencies in Vehicle Routing Problems (IVRP). The paper conceptualizes the priorities that arise in vehicle routing and stress the interconnectivity between priorities as it affects the interjectory intermittent situations and road restrictions. Solving problems of this nature could be quite tasking, requiring optimizing along with different directions. Reasons for these are associated with uncertainties that real-life situations make life dynamical, opening the vista that brought about intermittencies in Vehicle Routing Problems (IVRP). To achieve these feats, this paper formulates dynamics that fuse the various stages of LRC into ERC as well as considers the splitting effect caused by the road restrictions, analyses the fused LRC into the formulated relation, and encapsulates road restrictions.

Keywords: Road Restriction, Early Request Customers (ERC), Forced Split Delivery, Re-optimization, Late Request Customers (LRC), Intermittent Situation, Re-activation

1. Introduction

The classical Vehicle Routing Problems (VRP) are directed at determining a set of paths beginning at the depot and terminating at the depot, finding the minimum cost,

searching for the shortest path, minimizing the number of vehicles used in the delivery, etc., that enables the known request of all the nodes/customers to be satisfied. Every node/customer is only permitted to be serviced once by only one vehicle, and each vehicle has a finite carrying capacity. Economic globalization has brought about an increased

movement of commodities and services rendered all around the globe. The few commodities, services rendered, and resources transportation coupled with complex planning have increased the pressure on the cost through close competition among logistics service providers that make the use of computer-aided systems necessary for the proper planning and adequate scheduling of the transportation systems. A paramount sub-task evident in this phase is the operational planning of vehicles or other specialized means of transportation so involved. These numerous optimization enterprises are known and classified as VRPs.

Several thousands of publications on the variants of the VRP indicate its practical importance and empirical relevance of the NP-hard optimization problem to life. Therefore, several specified techniques to solve these variants of the VRP can be seen in the literature [1]. The drawback is that most of these techniques are inflexible and too specific and require a lot of effort to make the techniques conform to the slightest modified problems.

The notable developments in information technology and communication have made it possible for organizations to direct attention toward effectiveness, efficiency, robustness, and timeliness in the entire distribution process. With these, VRP with Time Window (VRPTW) has invariably turned out to be a priceless tool in directing different sections of the distribution design chain and its operations [2]. Remarkable significant applications of VRPTW to day-to-day activities include deliveries to supermarkets, industrial refuse collections, patrol service, school bus routing, security patrol, urban newspaper distribution, banks, and postal deliveries. The increased VRPTW practical visibility has shown tremendously in its widened parallel development and in-depth research focused on solving this class of problem.

Moreover, most real-life situations are more complex to handle than the similar problems itemized in literature and observe changes with time occurring among different customers, vehicles, and routes. To circumvent these issues, integrated modeling coupled with an optimization framework for tackling complex and practical relevant VRP that considers interdependencies in VRP, the inflow of priorities, and restrictions on the road to be traversed by the vehicle is in place. Formulations in literature present minimum traveling time problems with some constraints placed on them. Another traditional VRP variation is VRP with Split Delivery (VRPSD). In the classical VRPSD, the clientele can be visited by more than one vehicle [3]. Thus, for VRPSD, besides from considering the delivery paths and determining the amount to be supplied to each customer by each vehicle, we hope to stress forced split delivery caused by road restrictions. However, the fundamental objectives of VRP include determining the minimum number of vehicles at the station, the minimum time of travel of each vehicle, and the minimum costs of traversing the routes.

2. Formulation Requisites

Considering the vehicle routing for a given time, T . Let

$C = \{c_i: i = 0, 1, 2, \dots, N\}$ represent a set of N customers where c_0 is the depot. Let $V = \{v_k | k = 1, 2, 3, \dots, M\}$ represent the set of homogenous M vehicles positioned at the terminal, c_0 . By Russell, R. A. and Chiang, W. C. [4], (i, j) is the associated pair of locations with $i, j \leq N$ and the travel time, t_{ij} , from one customer c_i to another c_j and a distance traveled, $d(i, j) = d_{ij}$, that are symmetrical, i. e. $t_{ij} = t_{ji}$ and $d_{ij} = d_{ji}$.

According to Christofides, N. *et al.* [5], every customer, c_i , has the basic fundamental requirements of the quantity of the product that will be delivered by the vehicle, $Q(v_k)$, the duration t_{ij} , required by the vehicle moving from the terminal or from one customer to another customer to unlade the quantity moving to the next customer or go back to the terminal after visiting all the customers on that path or has exhausted all the quantities carried.

There is a set of identical vehicles, V such that the quantity that vehicle can carry, $v_k \in V$ is denoted by $Q(v_k)$. Like the customers having some set-aside requirements, the vehicles also have the following requirements [6] thus: the vehicle has a limited working duration, T_k , from the starting time, T_k^s , to the finishing time, T_k^f . The fixed cost, FC_k , is the salaries/wages of the drivers, and the loaders/unloaders attached to the vehicles are paid including the maintenance. The carrying capacity, $Q(v_k)$, of the vehicle is the maximum load the vehicle can carry at a time.

For Christofides, N. *et al.* [5] to model the problem, the following general assumptions as it concerns the customers' requirements and vehicles' characteristics were itemized: The variable cost, VC_{ij} , is the least path cost traversed by the vehicle from the customer c_i to the next customer c_j . The travel time, t_{ij} , is the corresponding duration of the vehicle spent from customer c_i to another customer c_j . The set $R_i = \{r_i(1), \dots, r_i(N)\}$ represents the of routes for the vehicle v_k , where $r_i(N)$ represents the n th customer while N represents the number of customers on that route. Likewise, it is assumed that every path must terminate at the depot hence, $r_i(N + 1) = 0$.

3. Road Restrictions

Transportation is a key to life and plays an essential role in our everyday day life activities. The evolvment and development of road congestions related to transport-energy consumption, and the adverse effects of transportation on the environment have drawn more and more concerns globally. With an increasing number of vehicular flows on the highway, it has led to traffic jams, pollution, road degradation, and lots more, traffic planners hence tend to put restrictions on highways ranging from one reason to the other in different ways. Therefore, continuous concerted efforts on VRP will not be directed towards reducing the amount spent on transportation only but, towards contributing to the general protection of our environment.

Some roads are more susceptible to damage than others based on poor drainages, weather conditions, and other variables. Road restrictions are mostly placed based on the

weather conditions and frost testing on some occasions. Road bans are effective tools for preventing road damage, reducing maintenance costs, and ensuring roadways remain safe for all motorists.

Road restrictions can be grouped as follows:

- 1) Road Time Restriction, $TR(v_k)$: The $TR(v_k)$ disallows the movement of vehicle at a particular time in some particular places and restricts movements of some types of vehicles on some routes to reduce the traffic in such areas.
- 2) Vehicular Weight Restriction, $WR(v_k)$: The $WR(v_k)$ is placed on some roads to regulate the weight of vehicles that traverse such roads purposely to keep the road infrastructure from further damage or total breakdown of the road [7].
- 3) Vehicular Height Restriction, $HR(v_k)$: Not all roads allow for any height of the vehicle. Height restrictions are placed to regulate either heavy duties vehicles or vehicles with too high consignment from plying a particular road to avoid degradation, total damage, or accident on the road.
- 4) Customer's Preference for a type of vehicle, $CP(v_k)$:

When a road ban occurs, signs indicating the allowed axle percentages are posted, and the ban is monitored and enforced to ensure compliance. Such restrictions are not for life rather, the authority in charge fixes them. Once the road has been fixed and determined to be structurally sound, load restrictions can be rescinded.

4. Forced Split Deliveries

Traditionally, the option of a split supply makes servicing of a clientele whose request exceeds the vehicle carrying quantity possible. The act of splitting may eventually lead to cost reduction. The VRP with time windows and split deliveries (VRPTWSD) according to Olateju, S. O. *et al.* [8], is an extension of the VRPSD, to which the time window constraints were added. Corberan, Á., and Laporte, G. [9] analyzed the complexity of the VRP and came up with the conclusion that in practice all VRPs are NP-hard (among them the classical vehicle routing problem) since they cannot be solved in polynomial time. According to Dror, M., and Trudeau, P. [10] the VRPTW is also NP-hard because it is an extension of the VRP. Although the VRPSD is a relaxed VRP, it is still NP-hard as stressed by Bräysy, O. and Gendreau, M.

$$q[c_1(v_k)] + q[c_2(v_k)] + \dots + q[c_{N-n}(v_k)] = \sum_{i=1}^{N-n} q[c_i(v_k)]. \quad (3)$$

Then, the quantity to be delivered by the vehicle v_k to the next customer c_{N-n+1} , is given by: $q[c_{N-n+1}(v_k)]$ and the total quantity to be delivered by vehicle v_k is given by:

$$\sum_{i=1}^{N-n} q[c_i(v_k)] + q[c_{N-n+1}(v_k)] = \sum_{i=1}^{N-n+1} q[c_i(v_k)]. \quad (4)$$

With (4), if the quantity $q[c_{N-n+1}(v_k)]$, which would have been delivered to the next customer, c_{N-n+1} , along the same route is not sufficient i.e.

$$\sum_{i=1}^{N-n+1} q[c_i(v_k)] > Q(v_k) \quad (5)$$

then, it occasioned a split delivery to be undertaken by another vehicle, v_{k+1} as:

$$\{\sum_{i=1}^{N-n} q[c_i(v_k)]\} + q[c_{N-n+1}(v_{k+1})] \quad (6)$$

[11] and Adebayo, K. J. [12]. Therefore, VRPTWSD is NP-hard, since it is a fusion of the VRPTW and VRPSD.

In this case, we want to direct our searchlight to VRP with Forced Split Delivery and make efforts toward its formulation herein. Since splitting requests a clientele to be satisfied by more than one vehicle, it occurs in most cases where the demand is higher than the capacity of the vehicle. However, we intend to redirect our attention to other causes of splitting as occasioned by road restrictions: Customers' Vehicle Preference, $CP(c_i)$, Road Time Restriction, $TR(v_k)$, Vehicle Weight Restriction, $WR(v_k)$, or Vehicle Height Restriction, $HR(v_k)$.

In real-life situations, irrespective of the types of vehicles used, split deliveries occur as long as the customer's demand cannot be entirely delivered by just a vehicle. Findings have shown that there are cases whereby the demand of customers outweighs the carrying capacity of the vehicles [8, 13]. Since by traditional design and formulation of VRP, a vehicle is not permitted to serve a customer twice the same day hence, necessitates split delivery.

The reasons for split deliveries include:

the quantity that the customer demands, $q(c_i)$, outweighs the carrying capacity of the vehicle, $Q(v_k)$, given by the relation:

$$q[c_i(v_k)] > Q(v_k) \quad (1)$$

This would lead to the delivery being done by more vehicles as:

$$SD_1(c_i) = Q(v_k) + Q(v_{k+1}) - q[c_i(v_k)] \geq q(c_i) \quad (2)$$

where $Q(v_k)$ and $Q(v_{k+1})$ are the carrying capacities of vehicles v_k and v_{k+1} respectively, $q[c_i(v_k)]$ is the amount supplied or to be supplied to customer c_i , $SD_1(c_i)$ is a split delivery.

the vehicle, v_k , has visited some clientele say, c_1, c_2, \dots, c_{N-n} , along the path, $r_i(S_i)$, with the respective quantities, $q(c_1), q(c_2), \dots, q(c_{N-n})$, where N is the number of customers in the system and $n < N$ represent the number of customers that have been visited along the path $r_i(S_i)$. A slight digression from the line of thought [8, 13] is what is obtainable in what follows as the quantities delivered by the vehicle v_k to the customers c_1, c_2, \dots, c_{N-n} , is given by:

From (6), the split quantity, $q[c_{N-n+1}(v_{k+1})]$ required by c_{N-n+1} is the remaining quantity to have been delivered by v_k which is:

$$Q(v_k) - \sum_{i=1}^{N-n} q[c_i(v_k)] + q[c_{N-n+1}] = q[c_{N-n+1}(v_{k+1})] \quad (7)$$

Since the amount of commodity that will be supplied by $Q(v_{k+1})$ cannot be determined a priori then:

$$SD_2(c_i) = Q(v_k) + Q(v_{k+1}) + \dots + Q(v_M) - \sum_{i=1}^{N-n} q[c_i(v_k)] = \sum_{k=1}^M Q[(v_k)] - \sum_{i=1}^{N-n} q[c_i(v_k)] \geq q[c_{N-n+1}] \quad (8)$$

From the above, a customer to whom the split delivery condition applies can only be linked to only one of (2) and (8).

5. Intermittencies in VRP

According to Adebayo, K. J. and Aderibigbe, F. M. [14] and Larsen, A. [15], the waiting customers before setting out of the vehicle from the depot are classified as Early Request Customers (ERC) with

$$ERC = \sum_{i=1}^N ERC_i \quad (9)$$

where N stands for the number of customers in the set. Let the anticipatory customers be referred to as Late Request Customers (LRC) with

$$LRC = \sum_{i=1}^x LRC_{N_S}^i \quad (10)$$

where the number of anticipatory customers is x , the customers already serviced before the LRC is to be served are presented by N_S and $c_{N_S}^i$ is the set of LRC.

Within the specified time frame, each vehicle is expected to get to the customers in a service location. The entire tour begins and ultimately ends at the depot. At least one ERC must be served first before any LRC and all the ERC in line must be served. In the event of the tour, a set of LRC may stochastically request service. The LRC remains not known until the dispatched manager makes their requests known. However, the time and location of the LRC are being guided by a known probability scheme.

Each time a vehicle sets out to service customers, the dispatcher decides which of the occurred requests' subset is to be designated to a particular vehicle and as long as the vehicle is still within the service period, a record on the entire tour is kept.

When a vehicle is assigned to the LRC, the vehicle is expected to service the remaining customer on that route within the time frame. The dispatcher aims at maximizing the number of LRC to which vehicle is assigned subject to the degree of dynamism. Depending on the number of ERC and their corresponding entering time, Mitrovic-Minic, S., *et al.* [16] suggested a potent means of determining the degree of dynamism. More so that the problem is intermittently dynamical, there is a need to adaptively restructure the existing service pattern to take care of the LRC to adhere to the priorities and road restrictions.

The most effective and efficient way to achieve this is to re-optimize part of the ERC solution then, the LRC is being inserted into the existing ERC solution process. For better planning, it is expedient to put into consideration anticipatory

customers for numerous reasons. Solving a dynamical pickup and delivery problem opined by Mitrovic-Minic, S., *et al.* [17] suggested a double-horizon heuristic that focuses on short-term goals by minimizing the total distance traveled and long-term goals by maximizing the slack time to accommodate servicing of the LRC. However, Branke, J., *et al.* [18], Pureza, V. and Laporte, G. [19] and Stewart, W., and Golden, B., Stewart, W., and Golden, B. [20] investigated the waiting strategies and improved on the solution by forcing the vehicles to wait at certain places just to buy time.

According to Larsen, A., *et al.* [21], the IVRP with stochastic requests varies in their levels of uncertainty. Particularly, these variations are on how many LRC might place an order within the time horizon when the ERC is to be serviced. Adebayo, K. J. and Aderibigbe, F. M. [14], Larsen, A. [15], and Ritzinger, U. *et al.* [22] referred to the uncertainty rate for the expected customers' requests as Degree of Dynamism and Dynamical Degree (DD) respectively. The dynamic for DD is defined as:

$$DD = \frac{LRC}{OC} \quad (11)$$

where the Overall Customers (OC), is the total numbers of ERC and LRC. The DD will be considered in three possible ways:

(i) The first kind is when the LRC comes just after all the ERC have been serviced as presented by Larsen, A. [15] The relations (9) and (10) thus give rise to:

$$OC = ERC + LRC = \sum_{i=1}^N c_i + \sum_{i=1}^x c_{N_S}^i \quad (12)$$

The case is usually easy to address compared to others categories in that, the initially planned ERC tour is not affected in any way. With or without the LRC in this case, all the ERC are kept unaltered, treated, and serviced first. From (11) and (12), the DD in this category is given by:

$$DD_1 = \frac{LRC}{OC} = \frac{LRC}{ERC+LRC} = \frac{\sum_{i=1}^x c_{N_S}^i}{\sum_{i=1}^N c_i + \sum_{i=1}^x c_{N_S}^i} \quad (13)$$

(ii) the Second kind is a case in which the LRC comes after a fractional part of the ERC, i.e. ERC_1 , has been serviced and the remaining ERC i.e. ERC_2 given by

$$ERC_2 = ERC - ERC_1 \quad (14)$$

are serviced after all possible LRC have been serviced thus:

$$OC = ERC_1 + LRC + ERC_2 \quad (15)$$

where $ERC_1 + ERC_2 = ERC$ and the resulting OC in the second case are given by:

$$OC = \sum_{i=1}^{G_1} c_i + \sum_{i=1}^x c_{N_S}^i + \sum_{i=G_1+1}^{N-G_1} c_i \quad (16)$$

where $G_1 < N$, represents the *ERC* that has been serviced after which the *LRC* request will be met and x represents the maximum possible *LRC* that can be taken into consideration such that the initial tour plan will not be affected.

Also, from (11) and (16), the *DD* in the second case is given by:

$$DD_2 = \frac{LRC}{OC} = \frac{LRC}{ERC_1 + LRC + ERC_2} \quad (17)$$

$$OC = ERC_1 + LRC_1 + ERC_2 + LRC_2 + \dots + LRC_y + ERC_M \quad (20)$$

where $M \leq (N - 1)$ is the possible number of *ERC* intermittencies serviced in the course of the servicing and $y \leq (x - 1)$ is the number of *LRC* intermittencies undertaken. With

$$ERC = ERC_1 + ERC_2 + ERC_3 + \dots + ERC_M = \sum_{i=1}^{G_1} c_i + \sum_{i=G_1+1}^{G_2=N-G_1} c_i + \sum_{i=G_2+1}^{G_3=N-G_2} c_i + \dots + \sum_{i=G_{y-1}+1}^{G_y=N-G_{y-1}} c_i + \sum_{i=G_y+1}^{N-G_y} c_i \quad (21)$$

and

$$LRC = LRC_1 + LRC_2 + \dots + LRC_y = \sum_{i=1}^{x_1} c_{G_1}^i + \sum_{i=1}^{x_2} c_{G_2}^i + \sum_{i=1}^{x_3} c_{G_3}^i + \dots + \sum_{i=1}^{x_3} c_{G_{n-1}}^i + \sum_{i=1}^{x_n} c_{G_n}^i \quad (22)$$

The resulting *OC* in the third case is thus:

$$OC = \sum_{i=1}^{G_1} c_i + \sum_{i=1}^{x_1} c_{G_1}^i + \sum_{i=G_1+1}^{G_2=N-G_1} c_i + \sum_{i=1}^{x_2} c_{G_2}^i + \sum_{i=G_2+1}^{G_3=N-G_2} c_i + \sum_{i=1}^{x_3} c_{G_3}^i + \dots + \sum_{i=1}^{x_n} c_{G_n}^i + \sum_{i=G_n+1}^{N-G_n} c_i \quad (23)$$

where $G_1, G_2, \dots, G_{n-1}, G_n$ represent the customers that had been serviced at the time the anticipatory customer's request comes in, and $G_1 + G_2 + \dots + G_n = N$. Hence, the *DD* for the third case from (11), (19), and (23) is given by:

$$DD = \frac{\sum_{i=1}^{x_1} c_{G_1}^i + \sum_{i=1}^{x_2} c_{G_2}^i + \sum_{i=1}^{x_3} c_{G_3}^i + \dots + \sum_{i=1}^{x_n} c_{G_n}^i}{\sum_{i=1}^{G_1} c_i + \sum_{i=1}^{x_1} c_{G_1}^i + \sum_{i=G_1+1}^{G_2=N-G_1} c_i + \sum_{i=1}^{x_2} c_{G_2}^i + \sum_{i=G_2+1}^{G_3=N-G_2} c_i + \sum_{i=1}^{x_3} c_{G_3}^i + \dots + \sum_{i=1}^{x_n} c_{G_n}^i + \sum_{i=G_n+1}^{N-G_n} c_i} \quad (24)$$

It is worthy of note that, in any of the three cases, the *LRC* cannot come ahead of the first *ERC*. The *DD* is a yardstick to classify IVRP into stochastic requests. From Ehmke, J. F. and Campbell, A. M. [23] and Maxwell, M. S. *et al.* [24], a moderate *DD* realized in practices includes distribution of oils, transportation of patients, and grocery deliveries. Its range of utilization with high-level *DD* entails emergency vehicles or courier services as Thomas, B. W. [25] and Goodson, J. C., *et al.* [26]. A high-level *DD* is mostly found in practical applications which include: responsive demand transportation, same-day delivery, and shared mobility as opined by Voccia, S. A., *et al.* [27], Brinkmann, J., *et al.* [28], and Ulmer, M. W. [29]. For a more detailed classification of DSVRP applications, see

Larsen, A. [15] and Miller, D. L., and Pekny, J. F. [30].

6. Objective Function Formulation

If the vehicle v_k visits the customer c_j immediately after servicing the customer c_i , then $\xi_{ijk} = 1$ otherwise, $\xi_{ijk} = 0$. As opined by Larsen, A. [15], a typical routing problem with multiple priorities is considered to be a multi-objective problem. Where $Min J_1$ computes the least path or distance carrying cost, $Min J_2$ computes the fixed cost, $Max J_3$ is targeted at evaluating the priorities and $Max J_4$ is set at calculating the *OC* which is the sum of the *ERC* and *LRC*. Hence,

$$Min J_1 = \alpha \sum_{i=1}^{N+x} \sum_{k=1}^M \left(\sum_{i=1}^{G_1} d_{ij}(c_i) + \sum_{i=1}^{x_1} d_{ij}(c_{G_1}^i) + \sum_{i=G_1+1}^{G_2=N-G_1} d_{ij}(c_i) + \sum_{i=1}^{x_2} d_{ij}(c_{G_2}^i) + \dots + \sum_{i=1}^{x_n} d_{ij}(c_{G_n}^i) + \sum_{i=G_n+1}^{N-G_n} d_{ij}(c_i) \right) \xi_{ijk} \quad (25)$$

$$Min J_2 = \beta \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^M FC(v_k) \xi_{ijk} \quad (26)$$

$$Max J_3 = \gamma \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^M \delta(c_i) \xi_{ijk} \quad (27)$$

$$Max J_4 = \sum_{i=1}^{N+x} \sum_{k=1}^M \left(\sum_{i=1}^{G_1} c_i + \sum_{i=1}^{x_1} c_{G_1}^i + \sum_{i=G_1+1}^{G_2=N-G_1} c_i + \sum_{i=1}^{x_2} c_{G_2}^i + \dots + \sum_{i=1}^{x_n} c_{G_n}^i + \sum_{i=G_n+1}^{N-G_n} c_i \right) \xi_{ijk} \quad (28)$$

where α, β , and γ by Kohl, N. *et al.* [31] are arbitrary constants for weighting the terms (25), (26), and (27) corresponding to each objective.

The Road restricted IVRP objective function with priorities to which this paper aimed at formulating is the one found on combining all the four objectives in (25), (26), (27), and (28) as:

$$J = \text{Min } J_1 + \text{Min } J_2 + \text{Max } J_3 + \text{Max } J_4 \quad (29)$$

$$J = \alpha \sum_{i=1}^{N+x} \sum_{k=1}^M \left(\sum_{i=1}^{G_1} d_{ij}(c_i) + \sum_{i=1}^{x_1} d_{ij}(c_{G_1}^i) + \sum_{i=G_1+1}^{G_2=N-G_1} d_{ij}(c_i) + \sum_{i=1}^{x_2} d_{ij}(c_{G_2}^i) + \dots + \sum_{i=1}^{x_n} d_{ij}(c_{G_n}^i) + \sum_{i=G_n+1}^{N-G_n} d_{ij}(c_i) \right) \xi_{ijk} + \beta \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^M FC(v_k) \xi_{ijk} + \gamma \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^M \delta(c_i) \xi_{ijk} + \sum_{i=1}^{N+x} \sum_{k=1}^M \left(\sum_{i=1}^{G_1} c_i + \sum_{i=1}^{x_1} c_{G_1}^i + \sum_{i=G_1+1}^{G_2=N-G_1} c_i + \sum_{i=1}^{x_2} c_{G_2}^i + \dots + \sum_{i=1}^{x_n} c_{G_n}^i + \sum_{i=G_n+1}^{N-G_n} c_i \right) \xi_{ijk} \quad (30)$$

Subject to:

$$\sum_{i=0}^N \sum_{j=1}^N \sum_{k=1}^M \xi_{ijk} \leq 1, j = 1, \dots, N \quad (31)$$

$$\sum_{i=0}^N \sum_{p=1}^N \sum_{k=1}^M \xi_{ipk} - \sum_{p=1}^N \sum_{j=2}^N \sum_{k=1}^M \xi_{pjk} \quad (32)$$

$$\sum_{i=0}^N \sum_{j=1}^N \sum_{k=1}^M q(c_i) \xi_{ijk} \leq Q(v_k) \quad k = 1, \dots, \quad (33)$$

$$\sum_{i=0}^N \sum_{j=1}^N \sum_{k=1}^M t_{ij} \xi_{ijk} \leq T_k^f - T_k^s, \quad (34)$$

$$\sum_{i=0}^N \sum_{j=1}^N \sum_{k=1}^M \xi_{ijk} \leq 1, \quad (35)$$

$$y_i - y_j + N \sum_{i=0}^N \sum_{j=1}^N \sum_{k=1}^M \xi_{ijk} \leq (N-1) \quad (36)$$

$$\xi_{ijk} \in \{0,1\} \quad \forall i, j, k \text{ and } p = 1, \dots, N \quad (37)$$

The constraint in (31) expresses that every customer can only be serviced once in a day by a vehicle. Constraint (32) stresses that any vehicle that services a particular customer ultimately must leave such a customer. Constraint (33) relates to the carrying capacity of the vehicle. Constraint (34) indicates the working time duration on each route. Constraint (35) stresses the use of a vehicle at most once a day. The relation (36) with y_i arbitrary, is the sub-tour-elimination term attached to Travelling Salesman Problem (TSP) by Kohl, N. *et al.* [31] and as stressed by Jean-Francois, C. *et al.* [32] in VRP. The sub-tour elimination makes sure that each vehicle passes through the depot. The constraint in (37) is the integrality conditions.

From the formulated IVRP objective above, should the IVRP aims to determine the priorities alone then, the series in (25), (26), and (28) are set as zero in (30). If the IVRP is aimed at determining the priorities and the costs then, the series (28) will be set as zero. When the target is to calculate the intermittencies, (25), (26), and (27) are set at zero in (30) but, if the aim is to compute the variable cost, fixed cost, the priorities, and the intermittencies then, (30) holds.

However, the central idea behind the IVRP is to assist the dispatcher manager to plan the distribution/collection network ahead of time such that a customer gets the desired quantity and is delivered at the said time. It enables timely delivery, vehicle space, and capacity management of the vehicles. With these, it ensures that servicing of customers is based on the priorities such customers earlier set with a view to minimizing both the fixed and variable costs and maximizing the profit. With a proviso that, should an intermittency customer come in between the ERC, such customers' requests are also met without affecting the earlier planned routes, timing, and quantities.

7. Conclusion

Real-life situations that are characterized by changes on daily basis have made IVRP with multiple priorities inevitable. As such, rather than losing customers to any close competitors, more customers will be won hence, increasing the profit margin. The advent and improvement in information technology have greatly contributed to the solution to this class of problem. It has become less difficult due to the use of network facilities, a Global System for Mobile communication, and a Global Positioning System. Otherwise, its attainment would have been a mirage and not feasible.

The vehicle must carry along with it an anticipatory quantity and create room for additional anticipatory time to cover the supply and delivery. There should be information inter-connectivity from the depot to customers via the vehicles in the chain.

While investigating IVRP with multiple priorities, randomly generated data were used against real-life data. Reasons for this are connected to: firstly, data randomly generated often enables an in-depth analysis. This is because the sets of data can be constructed such that other issues can be taken care of alongside. Secondly, most real-life IVRP with multiple priorities do not capture all the data needed for holistic analyses of the routing problem. The full information about the geographical locations of all the vehicles not known at the time the LRC request is received is one of the missing data items in our day-to-day business activities hence, necessitating randomly generated data.

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