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Graph Colouring to Solve Both Balanced and Unbalanced Transportation Problems

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Abstract: Transportation is one of the various worldwide challenges that organizations must tackle. The Transportation Problem (TP) is also an important topic in the subject of optimization, where the aim is to reduce the overall transportation cost of distributing from a particular number of sources to a specific number of destinations (locations). This paper aims to investigate the most effective way to solve the Transportation Problem (TP) using the line (edge) colouring of a bipartite network. To solve the TP, several solutions have been proposed in the literature. The Initial Basic Feasible Solution (IBFS) and the Optimal Solution are the two solutions of TP. The North-West Corner Method (NWCM), the Lowest Cost Method (LCM), Row Minima Method (RMM), Column Minima Method (CMM), and Vogel's Approximation Method (VAM) may be used to find an IBFS, while the Modified Distribution (MODI) Method and the Stepping Stone Method can be used to find an optimal solution for the TP In this paper, propose a new algorithmic technique that utilizes the line (edge) colouring of a bipartite network to obtain an optimal or nearly optimal solution to the TP. The proposed technique is applied to balanced and unbalanced TP and compared to other existing methods. Experimental results show that the line (edge) colouring algorithm requires fewer iterations to achieve optimality compared to other current methodologies. In conclusion, the line (edge) colouring algorithm is a highly effective method for solving TP. By representing the TP as a bipartite network and using the proposed algorithmic technique, the optimum or nearly optimum TP solution can be obtained quickly and efficiently. This approach has significant potential for optimizing transportation logistics in various industries.

Keywords: Column Minima Method, Graph Colouring, Modified Distribution, Optimization, Transportation Problem

1. Introduction

One of the interesting parts of operations research is the transportation problem (TP). When the sources and destinations are known and the demand and supply are met, industries need to reduce transportation costs. One of the best studies that apply to a wide range of decision problems that can be described as network optimization problems and addressed effectively and efficiently is the study of network models. The TP, the min-cost flow problems, and the max flow problems are all members of the family of network optimization problems. A network of edges and vertices may be used to describe these issues succinctly. The two main elementary application areas of mathematics are transportation networks and graph theory.

There are two types of TPs, namely balanced and unbalanced. A balanced TP occurs when the total supply of goods from sources is equal to the total demand for goods at destinations. In contrast, an unbalanced TP occurs when the total supply and demand are not equal. In such cases, a dummy source or destination can be added to balance the total supply and demand. The foundations of topological characteristics in graph theory were initiated and established by Antonievella [1] in 2014. Thus, Vimala and Kalpana [17] created the Bipartite Graph idea and used it in Matching and Colouring. Hitchcock [16] initially developed the TP, and Koopmans [33] later created the Optimum Utilization of the Transportation System. Following that, Dantzig [4] and Charnes and Cooper [13] created the transportation Simplex Approach and the stepping stone method, respectively, to address this problem. Additionally, several heuristic approaches, including the Northwest Corner Method, the least cost method [15], Vogel's approximation method (VAM) [20, 22, 30], E. M. U. S. B. Ekanayake's methods [6, 9 – 13], K. P. O. Niluminda's methods [24 – 29], E. M. D. B. Ekanayake's methods [7, 8], and etc were suggested to find an Initial Basic Feasible Solution (IBFS) to the TP. Furthermore, T. Geetha and N. Anandhi [14] proposed a new method for solving unbalanced TP using standard deviations, Md Sharif Uddin [34] proposed an Improved Least-Cost Method to obtain a better IBFS to the TP, Dhia Abdul Sattar Kadhem [30] find IBFS of TP using maximum supplies and demands method, A. Edward Samuel Ramanujan [5] proposed IZPM – (Improved Zero Point Method) for the TPs, and Smita Sood and Keerti Jain [32] introduced a new method to find IBFS of TP called the maximum difference method. Moreover, M. Afwat [2], Lakhveer Kaur [3], Kirti Kumar Jain [18], T. Karthy and K. Ganesan [21], A. M. S. Juman [19, 20, 31] proposed several methods to solve TPs.

The primary objective of this study is to address the challenge of solving both balanced and unbalanced transportation problems (TPs) by utilizing bipartite network line colouring to achieve convergence and potentially identify an optimum solution. To achieve this objective, we have developed a heuristic method that is easy to implement and can efficiently find solutions to complex TPs. In the following sections, we will describe in detail the methods and procedures used in our study, which are designed to measure and evaluate the effectiveness of our proposed method. We will also provide illustrative examples to demonstrate the applicability of our method and compare its performance with existing methods. By doing so, we hope to contribute to the growing body of research in the field of TP optimization and provide a useful tool for practitioners in various industries.

2. Methods

This section describes some basic definitions, the mathematical formulation of TPs, and the research method used. In a TP, the points from which commodities are delivered to the requiring points are referred to as sources$(S_1, S_2, \ldots, S_m)$. The destination nodes in a TP, or the locations where commodities are delivered from the sources, are known as the demanding nodes $(D_1, D_2, \ldots D_n)$. The supply limit of a source is the amount of a commodity that is required to satisfy the demands of the demand nodes. Demand requirement is the term used to describe the quantity of a commodity needed to satisfy the needs of a demanding node. There are two types of TPs. Such as balanced TPs and unbalanced TPs. If the overall supply and demand are equal, a transportation problem is said to be balanced i. e. $Total\ Supply\ (\sum_{i=1}^{m} a_i) = Total\ Demand\ (\sum_{j=1}^{n} b_j)$ and if the overall supply and demand are not equal, a transportation problem is said to be unbalanced. i. e. $Total\ Supply\ (\sum_{i=1}^{m} a_i) \neq Total\ Demand\ (\sum_{j=1}^{n} b_j)$. A bipartite network (also known as a bi-graph) is a graph whose vertices may be split into two separate and independent sets, U and V; each edge links a vertex in U to a vertex in V. Typically, the components of the graph are referred to as vertex sets U and V. According to graph theory, line colouring refers to the process of assigning "colours" to a graph's lines so that no two incident lines have the same colour.

2.1. Mathematical Formulation of the Transportation Problem

Consider a scenario in which a specific commodity is typically produced at m production plants, known as sources, denoted by $S_1, S_2, \ldots S_m$ with respective capacities of $a_1, a_2, \ldots, a_m$, and delivered to n distribution centres, defined as destinations, represented by $D_1, D_2, \ldots D_n$ with respective demands $b_1, b_2, \ldots, b_n$. Assume also that the volume sent and the transit expense from the i^{th} source to the j^{th} destination is x_{ij} and c_{ij} respectively.

Where $i = 1, 2, \ldots, m$ & $j = 1, 2, \ldots, n$.

Mathematical Simulation of TP:

$$\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0; \forall\ i = 1, 2, \ldots, m \ \&\ j = 1, 2, \ldots, n$$

2.2. The Proposed Method

Both balanced and unbalanced TPs can be solved using the suggested approach.

Step 1: Create the Transportation Table (TT).

Step 2: Balance TT by adding a dummy row or dummy column if it is out of balance.

Step 3: Utilizing the transportation costs (c_{ij}) from the i^{th} source to the j^{th} destination draw the associated transportation network, and label the edges.

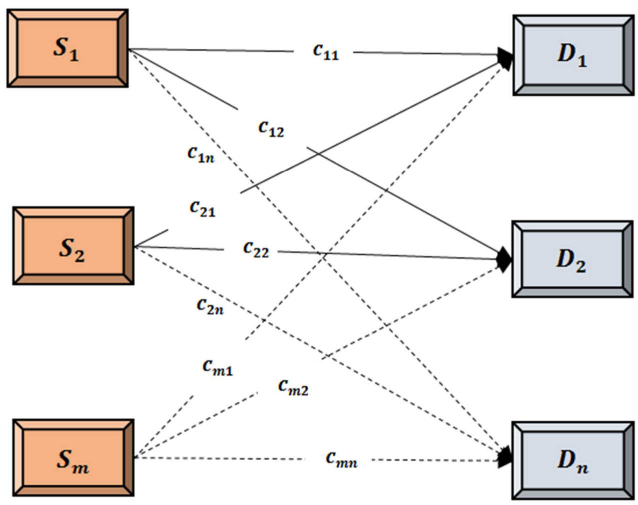


Figure 1. *Transportation Network.*

Step 4: Choose the edge that has the lowest c_{ij} from the i^{th} source to the j^{th} destinations (without c_{ij} values of dummy row or column), and then colour them all the same colour, let's assume "green".

Step 5: Choose the edge that has the lowest c_{ij} from the i^{th} sources to the j^{th} destination (without c_{ij} values of dummy row or column), and then colour them all the same colour, let's assume "purple".

Step 6: Adjust the supply and demand of the chosen cell supply and demand in TT, and then cross off the satisfied row or column by selecting the edge that is coloured in both colours.

Step 7: Update the TT in increasing order of the c_{ij} value if the network has more than one double-coloured edge.

Step 8: To satisfy all supply and demand, redraft the transportation network without crossing out supply or demand nodes.

Step 9: After all allocations are complete, choose the double-coloured edges with a zero c_{ij} value to adjust the supply and demand of TT if all supply and demand in the unbalanced problem are not fulfilled.

Step 10: Calculate the minimum total transportation cost of TT by the sum of the product of cost (c_{ij}) and the corresponding allocated value (x_{ij}). i. e. $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij}$

3. Results and Discussion

In this section, solve some illustrative examples using the proposed method and compare results with other existing methods.

3.1. Illustrative Examples

The newly proposed method applies to both balance and unbalanced TPs and compares them with an optimal solution in this section.

3.1.1. BTP – 1 (Balance TP) [11]

Table 1 shows the TP with three sources and four destinations. Total supply is equal to total demand. Therefore this is balanced TP. The proposed method is described in the following tables.

Step 1 & 2:

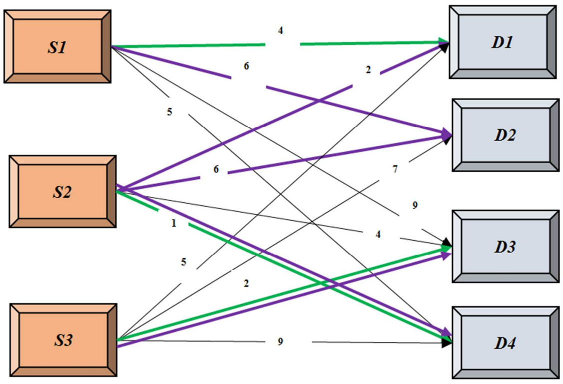
Table 1. *Initial Transportation Table.*

	D1	D2	D3	D4	Supply
S1	4	6	9	5	16
S2	2	6	4	1	12
S3	5	7	2	9	15
Demand	12	14	9	8	

Table 2. *Transportation Table_1.1.*

	D1	D2	D3	D4	Supply
S1	4	6	9	5	16
S2	2	6	4	(8)*1	12,4
S3	5	7	(9)*2	9	15,6
Demand	12	14	9,0	8,0	

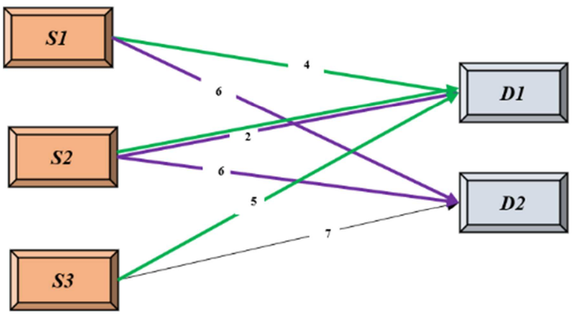
Steps 3 – 8:



- Select S2 – D4
- Select S3 – D3

Figure 2. *Transportation network_1.1.*

Figure 2 shows the transportation network of the given problem and the costs (c_{ij}) from the i^{th} source to the j^{th} destination represented in edge weights.



- Select S2 – D1

Figure 3. *Transportation network_1.2.*

Table 3. *Transportation Table_1.2.*

	D1	D2	D3	D4	Supply
S1	4	6	9	5	16
S2	(4)*2	6	4	(8)*1	12,4,0
S3	5	7	(9)*2	9	15,6
Demand	12,8	14	9,0	8,0	

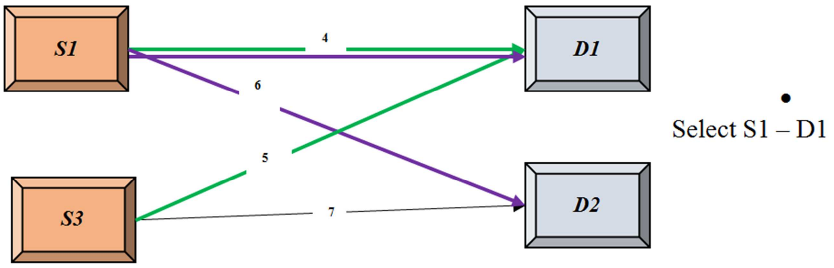


Figure 4. *Transportation network_1.3.*

Table 4. *Transportation Table_1.3.*

	D1	D2	D3	D4	Supply
S1	(8)*4	6	9	5	16,8
S2	(4)*2	6	4	(8)*1	12,4,0
S3	5	7	(9)*2	9	15,6
Demand	12,8,0	14	9,0	8,0	

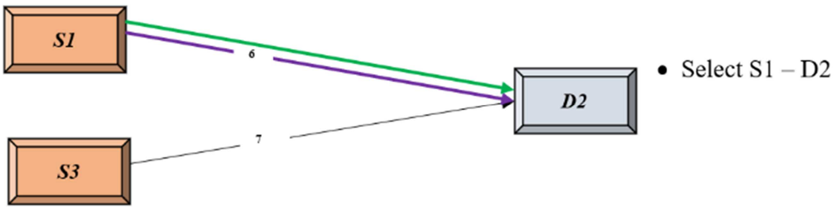


Figure 5. *Transportation network_1.4.*

Table 5. *Transportation Table_1.4.*

	D1	D2	D3	D4	Supply
S1	(8)*4	(8)*6	9	5	16,8,0
S2	(4)*2	6	4	(8)*1	12,4,0
S3	5	7	(9)*2	9	15,6
Demand	12,8,0	14,6	9,0	8,0	

Figure 6. *Transportation network_1.5.*

The final transportation table with all allocations is shown in below Table 6.

Table 6. *Transportation Table_1.5.*

	D1	D2	D3	D4	Supply
S1	(8)*4	(8)*6	9	5	16,8,0
S2	(4)*2	6	4	(8)*1	12,4,0
S3	5	(6)*7	(9)*2	9	15,6,0
Demand	12,8,0	14,6,0	9,0	8,0	

Step 10:

Total Transportation Cost

$$=(8\times4)+(8\times6)+(4\times2)+(8\times1)+(6\times7)+(9\times2)=156$$

The total transportation cost of this balanced TP is equal to 156. The VAM method solution and an optimal solution are also 156. Therefore, the proposed methods give an optimal solution in this balanced TP.

3.1.2. UBTP – 1 (Unbalanced TP) [14]

Table 7 shows the TP with three sources and four destinations. Total supply is not equal to total demand. Therefore, this is an unbalanced TP. By adding a dummy column this unbalanced TP can convert to a balanced TP. The balanced TP table is shown in Table 8. The proposed method is described in the following tables.

Step 1:

Table 7. *Initial Unbalanced Transportation Table.*

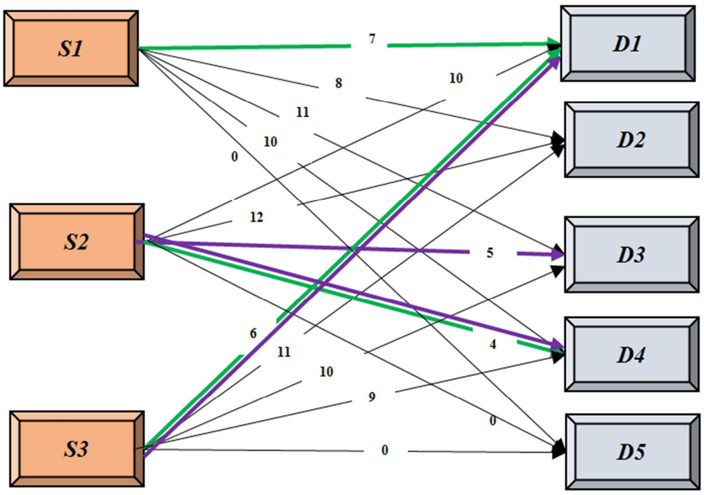
	D1	D2	D3	D4	Supply
S1	7	8	11	10	30
S2	10	12	5	4	45
S3	6	11	10	9	35
Demand	20	28	19	33	

Table 8. *Initial balanced Transportation Table.*

	D1	D2	D3	D4	D5 (Dummy)	Supply
S1	7	8	11	10	0	30
S2	10	12	5	4	0	45
S3	6	11	10	9	0	35
Demand	20	28	19	33	10	

Step 2:

Steps 3 – 8:



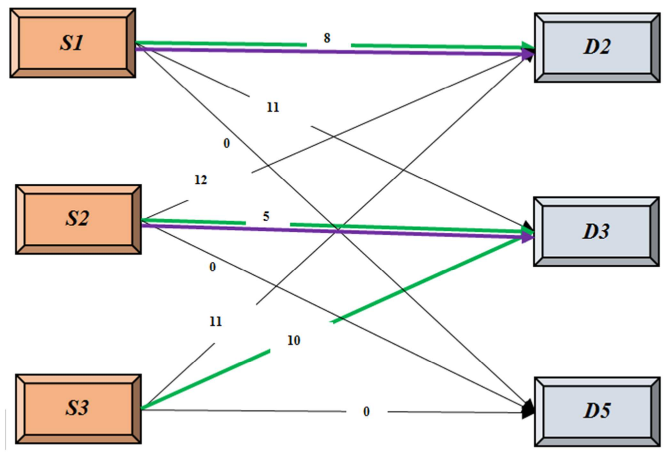
- Select S2 – D4
- Select S3 – D1

Figure 7. *Transportation network_2.1.*

Figure 7 shows the transportation network of the given problem and the costs (c_{ij}) from the i^{th} source to the j^{th} destination represented in edge weights. D5 is the dummy destination.

Table 9. *Transportation Table_2.1.*

	D1	D2	D3	D4	D5 (Dummy)	Supply
S1	7	8	11	10	0	30
S2	10	12	5	(33)*4	0	45,12
S3	(20)*6	11	10	9	0	35,15
Demand	20,0	28	19	33,0	10	



- Select S2 – D3
- Select S1 – D2

Figure 8. *Transportation network_2.2.*

Table 10. *Transportation Table_2.2.*

	D1	D2	D3	D4	D5 (Dummy)	Supply
S1	7	(28)*8	11	10	0	30,2
S2	10	12	(12)*5	(33)*4	0	45,12,0
S3	(20)*6	11	10	9	0	35,15
Demand	20,0	28,0	19,7	33,0	10	

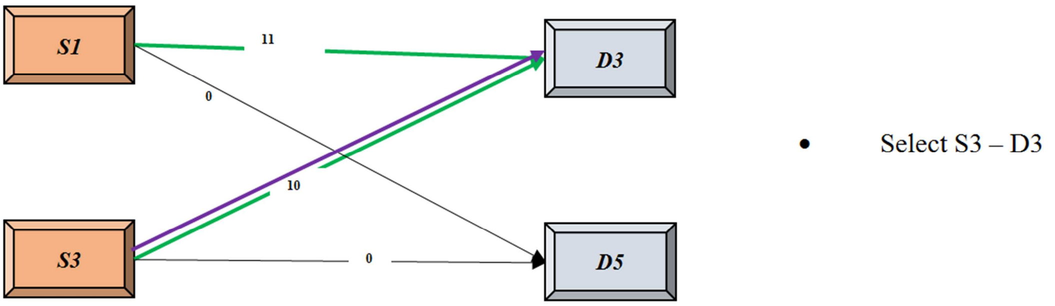


Figure 9. *Transportation network_2.3.*

Table 11. *Transportation Table_2.3.*

	D1	D2	D3	D4	D5 (Dummy)	Supply
S1	7	(28)*8	11	10	0	30,2
S2	10	12	(12)*5	(33)*4	0	45,12,0
S3	(20)*6	11	(7)*10	9	0	35,15,8
Demand	20,0	28,0	19,7,0	33,0	10	

Step 9:

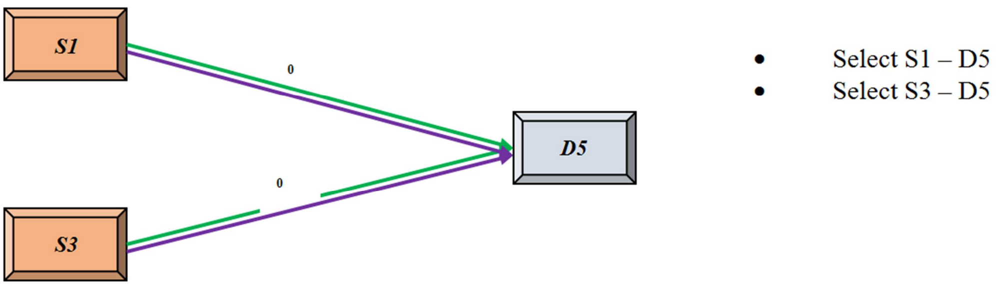


Figure 10. *Transportation network_2.4.*

The final transportation table with all allocations shows in below Table 12.

Table 12. *Transportation Table_2.4.*

	D1	D2	D3	D4	D5 (Dummy)	Supply
S1	7	(28)*8	11	10	(2)*0	30,2,0
S2	10	12	(12)*5	(33)*4	0	45,12,0
S3	(20)*6	11	(7)*10	9	(8)*0	35,15,8,0
Demand	20,0	28,0	19,7,0	33,0	10,8,0	

Step 10:

$$\text{Total Transportation cost} = (28\times8)+(2\times0)+(12\times5)+(33\times4)+(20\times6)+(7\times10)+(8\times0)=606$$

The total transportation cost of this unbalanced TP is equal to 606. The VAM method solution is 630 and the optimal solution is 606. Therefore, the proposed methods give an optimal solution for this unbalanced TP. By comparing it to the VAM method we can get a better solution.

3.2. Discussion

The efficiency of the suggested strategy is evaluated in this study by comparing the results. In Table 13 [11], the numerical data of balanced TPs from Table 14 are presented in detail.

Table 13. *The numerical data of balanced TPs.*

Problem (Balanced TP) [11]	Data of the problem
BTP_01	$c_{ij} = [4,6,9,5; 2,6,4,1; 5,7,2,9]$, $s_i = [16,12,15]$, $d_j = [12,14,9,8]$
BTP_02	$c_{ij} = [4,1,2,4,4; 2,3,2,2,3; 3,5,2,4,4]$, $s_i = [60, 35, 40]$, $d_j = [22, 45, 20, 18, 30]$
BTP_03	$c_{ij} = [5, 7, 10, 5, 3; 8, 6, 9, 12, 14; 10, 9, 8, 10, 15]$, $s_i = [5, 10, 10]$, $d_j = [3, 3, 10, 5, 4]$
BTP_04	$c_{ij} = [4, 3, 5; 6, 5, 4; 8, 10, 7]$, $s_i = [90, 80, 100]$, $d_j = [70, 120, 80]$
BTP_05	$c_{ij} = [6, 8, 10; 7, 11, 11; 4, 5, 12]$, $s_i = [150, 175, 275]$, $d_j = [200, 100, 300]$
BTP_06	$c_{ij} = [10, 2, 20, 11; 12, 7, 9, 20; 4, 14, 16, 18]$, $s_i = [15, 25, 10]$, $d_j = [5, 15, 15, 15]$

Problem (Balanced TP) [11]	Data of the problem
BTP_07	$c_{ij} = [50, 60, 100, 50; 80, 40, 70, 50; 90, 70, 30, 50]$, $s_i = [20, 38, 16]$, $d_j = [10, 18, 22, 24]$
BTP_08	$c_{ij} = [3, 1, 7, 4; 2, 6, 5, 9; 8, 3, 3, 2]$, $s_i = [300, 400, 500]$, $d_j = [250, 350, 400, 200]$
BTP_09	$c_{ij} = [9, 12, 9, 6, 9, 10; 7, 3, 7, 7, 5, 5; 6, 5, 9, 11, 3, 1; 6, 8, 11, 2, 2, 10]$, $s_i = [5, 6, 2, 9]$, $d_j = [4, 4, 6, 2, 4, 2]$
BTP_10	$c_{ij} = [6, 4, 1; 3, 8, 7; 4, 4, 2]$, $s_i = [50, 40, 60]$, $d_j = [20, 95, 35]$

We present performance comparisons between several well-known approaches, including the North-West Conner Method (NWCM), Least Cost Method (LCM), Row Minima Method (RMM), Column Minima Method (CMM), and Vogel's Approximation Method (VAM).

Table 14. *Comparative results of NWCM, LCM, RMM, CMM, VAM and New method (NEW) for 10 balanced TPs.*

Problem No	NWCM	LCM	RMM	CMM	VAM	NEW	Optimal
BTP_01	226	156	204	190	156	156	156
BTP_02	363	295	295	295	290	290	290
BTP_03	234	191	186	215	187	187	183
BTP_04	1500	1450	1450	1500	1500	1450	1390
BTP_05	5925	4550	5225	4550	5125	4550	4525
BTP_06	520	475	505	475	475	475	435
BTP_07	4160	3500	4120	3320	3320	3320	3320
BTP_08	4400	2900	2850	3600	2850	2850	2850
BTP_09	139	103	139	110	103	103	103
BTP_10	730	555	555	595	555	555	555

Figure 11 shows the comparison data from Table 14 as well as bar graphs that show the outcomes.

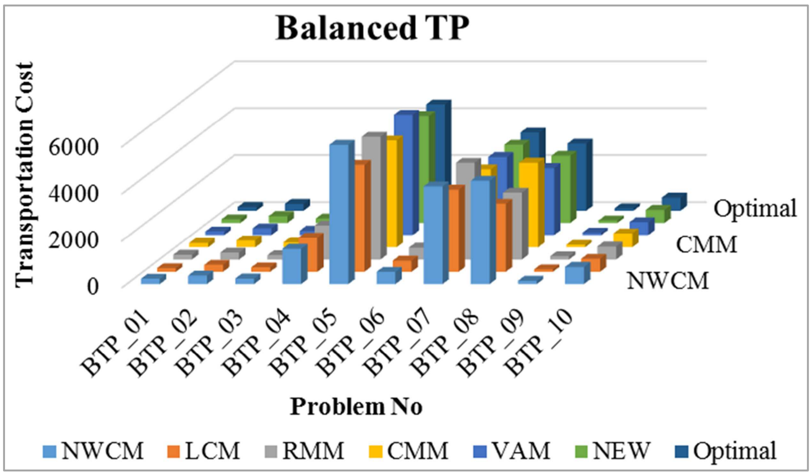


Figure 11. *Graphical representation of the comparative analyzed balanced TPs outcomes.*

By looking at above Table 14 and Figure 11, the proposed method is more efficient than other methods. We can get optimal solution 6 out of 10 balanced TP and near-optimal solution 4 out of 10 balanced TP. In BTP_04 and BTP_05, we can get a better solution using the proposed method than VAM. In Table 15, the numerical data of unbalanced TPs from Table 16 are presented in detail.

Table 15. *The numerical data of unbalanced TPs.*

Problem (Unbalanced TP)	Data of the problem
UBTP_01 [14]	$c_{ij} = [7, 8, 11, 10; 10, 12, 5, 4; 6, 11, 10, 9]$, $s_i = [30, 45, 35]$, $d_j = [20, 28, 19, 33]$
UBTP_02 [23]	$c_{ij} = [3, 4, 6; 7, 3, 8; 6, 4, 5; 7, 5, 2]$, $s_i = [100, 80, 90, 120]$, $d_j = [110, 110, 60]$
UBTP_03 [14]	$c_{ij} = [10, 15, 12, 12; 8, 10, 11, 9; 11, 12, 13, 10]$, $s_i = [200, 150, 120]$, $d_j = [140, 120, 80, 220]$
UBTP_04 [14]	$c_{ij} = [5, 6, 9; 3, 5, 10; 6, 7, 6; 6, 4, 10;]$, $s_i = [100, 75, 50, 75]$, $d_j = [70, 80, 120]$
UBTP_05 [14]	$c_{ij} = [6, 1, 9, 3; 11, 5, 2, 8; 10, 12, 4, 7;]$, $s_i = [70, 55, 70]$, $d_j = [85, 35, 50, 45]$
UBTP_06 [13]	$c_{ij} = [5, 4, 8, 6, 5; 4, 5, 4, 3, 2; 3, 6, 5, 8, 4;]$, $s_i = [600, 400, 1000]$, $d_j = [450, 400, 200, 250, 300]$

Problem (Unbalanced TP)	Data of the problem
UBTP_07 [13]	$c_{ij} = [12, 10, 6, 13; 19, 8, 16, 25; 17, 15, 15, 20; 23, 22, 26, 12]$, $s_i = [150, 500, 600, 225], d_j = [300, 500, 75, 100]$
UBTP_08 [13]	$c_{ij} = [5, 8, 6, 6, 3; 4, 7, 7, 6, 5; 8, 4, 6, 6, 4]$, $s_i = [800, 500, 900,], d_j = [400, 400, 500, 400]$
UBTP_09 [13]	$c_{ij} = [10, 8, 4, 3; 12, 14, 20, 2; 6, 9, 23, 25]$, $s_i = [500, 400, 300], d_j = [250, 350, 600, 150]$
UBTP_10 [13]	$c_{ij} = [6, 10, 14; 12, 19, 21; 15, 14, 17]$, $s_i = [50, 50, 50], d_j = [30, 40, 55,]$

We present performance comparisons between several well-known approaches, including the NWCM, LCM, RMM, CMM, and VAM.

Table 16. *Comparative results of NWCM, LCM, RMM, CMM, VAM and New method (NEW) for 10 unbalanced TPs.*

Problem No	NWCM	LCM	RMM	CMM	VAM	NEW	Optimal
UBTP_01	788	606	710	606	630	606	606
UBTP_02	1010	840	1410	840	880	840	840
UBTP_03	5070	4900	4770	5040	5020	4900	4720
UBTP_04	1780	1465	1745	1465	1555	1465	1465
UBTP_05	1125	965	965	1010	965	965	960
UBTP_06	8150	5750	6450	5850	6000	5750	5600
UBTP_07	12825	10375	14625	10675	11125	10375	10375
UBTP_08	13100	9200	9200	10300	9200	9200	9200
UBTP_09	18800	7750	11050	16900	8350	7750	7750
UBTP_10	1815	1695	1925	1695	1745	1695	1650

Figure 12 shows the comparison data from Table 16 as well as bar graphs that show the outcomes. By observing Table 16 and Figure 12, can get an optimal solution of 6 out of 10 unbalanced TP and a near-optimal solution of 4 out of 10 balanced TP. In UBTP_03, UBTP_06, and UBTP_10, we can get a better solution using the proposed method than VAM.

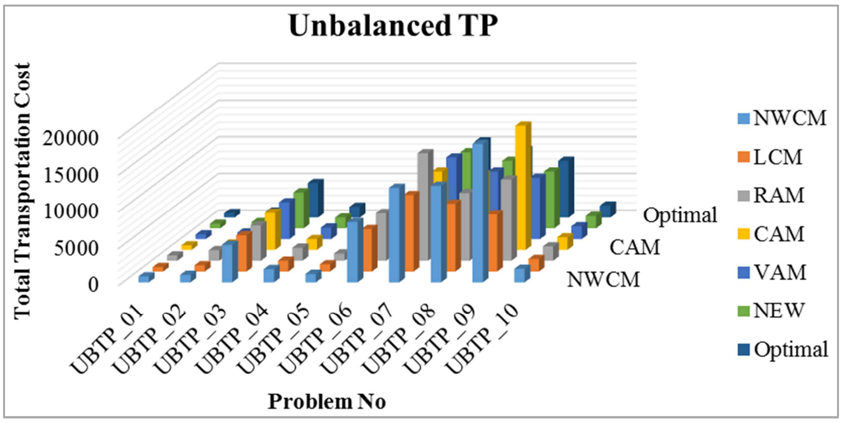


Figure 12. *Graphical representation of the comparative analyzed unbalanced TPs outcomes.*

4. Conclusion

This research proposes a novel algorithmic technique that uses line colouring of bipartite networks to solve both balanced and unbalanced Transportation Problems (TPs). The results of the study demonstrate that the proposed method can provide optimal or nearly optimal solutions to TPs with a high degree of success. The method was tested on 20 TPs, comprising 10 balanced TPs and 10 unbalanced TPs, and produced optimal solutions in 12 out of 20 problems, and near-optimal solutions in the remaining 8 problems. Notably, the near-optimal solutions obtained using the proposed method were better than those obtained using Vogel's Approximation Method (VAM).

The effectiveness of the proposed method is attributed to its simplicity and efficiency, which make it a promising approach for solving TPs in various fields, including logistics, supply chain management, and transportation engineering. The proposed method leverages line colouring of bipartite networks to generate an Initial Basic Feasible Solution (IBFS), which is then optimized through a limited number of iterations. The comparative analysis shows that the proposed method outperforms other established methods such as VAM, which is widely used in practice.

In conclusion, the proposed method represents a significant contribution to the development of efficient and innovative methods for solving TPs. The method's effectiveness, simplicity, and potential for real-world implementation make it a promising approach for solving TPs in various fields. Future research can be done to explore its effectiveness in more complex transportation scenarios and its potential for integration into transportation management systems.

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